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pün

$$1) A = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 1 \\ -2 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 1 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 1 \\ 0 & -3 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{rank } A = 2$$

$$S_2 = \begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} = 3 \neq 0$$

$$S_{2+1} = \begin{vmatrix} -2 & 1 & 5 \\ 1 & -2 & -2 \\ 1 & 1 & -3 \end{vmatrix} = -12 - 2 + 5 + 10 - 4 + 3 = 0$$

C020m vorder.

$$-2x + y = 5 - t, \quad z = t$$

$$x - 2y = -2 - t$$

$$x = \frac{\begin{vmatrix} 5-t & 1 \\ -2-t & -2 \end{vmatrix}}{3} = \frac{-10 + 2t + 2 + t}{3} = \frac{3t - 8}{3}$$

$$y = \frac{\begin{vmatrix} -2 & 5-t \\ 1 & -2-t \end{vmatrix}}{3} = \frac{4 + 2t - 5 + t}{3} = \frac{3t - 1}{3}$$

$$z = t$$

t=0 tun kontrol

$$x = -\frac{8}{3} \quad y = \frac{1}{3} \quad z = 0$$

$$\frac{16}{3} - \frac{1}{3} = 5 = 5 \quad \checkmark$$

$$-\frac{8}{3} + \frac{2}{3} = -2 = -2 \quad \checkmark$$

$$-\frac{8}{3} - \frac{1}{3} = -3 = -3 \quad \checkmark$$



2) 29 puan

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ -1 & 2 & 1 \end{bmatrix}$$

$$A = LU$$

$$A \stackrel{\Sigma_1}{\sim} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -2 \\ 0 & 4 & 4 \end{bmatrix} \stackrel{\Sigma_2}{\sim} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -2 \\ 0 & 0 & \frac{12}{5} \end{bmatrix} = U$$

$$\Sigma_2: \alpha_3 \rightarrow \alpha_3 + \frac{4}{5}\alpha_2$$

$$A \stackrel{N}{\sim} \begin{bmatrix} 1 & 0 & 0 \\ 2 & -5 & -2 \\ -1 & 4 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & -2 \\ -1 & -\frac{4}{5} & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -1 & -\frac{4}{5} & \frac{12}{5} \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -1 & -\frac{4}{5} & 1 \end{bmatrix} = L$$

$A = LU$  Kontrol:

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -1 & -\frac{4}{5} & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -2 \\ 0 & 0 & \frac{12}{5} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ -1 & 2 & 1 \end{bmatrix} = A$$

3) 29 puan

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ -1 & 3 & 4 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -2 & 3 & -2 & 1 & 0 \\ 0 & 4 & 4 & 1 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 3 & -1 & 1 & 0 \\ 0 & -1 & 3 & -2 & 1 & 0 \\ 0 & 0 & 16 & -7 & 4 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 & -1 & 1 & 0 \\ 0 & 1 & -3 & 2 & -1 & 0 \\ 0 & 0 & 16 & -7 & 4 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & \frac{5}{16} & \frac{1}{4} & -\frac{3}{16} \\ 0 & 1 & 0 & \frac{11}{16} & -\frac{1}{4} & \frac{3}{16} \\ 0 & 0 & 1 & -\frac{7}{16} & \frac{1}{4} & \frac{1}{16} \end{bmatrix}$$

$A^{-1}$

$A \cdot A^{-1} = A^{-1} \cdot A = I$  Kontrol ?



a) Elemanter matris birim matrise bir tek elemanter işlem uygulanarak elde edilen matristir denir;

a)  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$   $\Sigma: x_1 \rightarrow x_1 + x_2$   $E = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$  elemanter matris

tersi için  $\Sigma^{-1}: x_1 \rightarrow x_1 - x_2$   $E^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

a) b)  $A = \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{2 \times 1}$   $B = \begin{bmatrix} 3 & 2 \end{bmatrix}_{1 \times 2}$

$$AB = \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{2 \times 1} \begin{bmatrix} 3 & 2 \end{bmatrix}_{1 \times 2} = \begin{bmatrix} 3 & 2 \\ 0 & 0 \end{bmatrix}_{2 \times 2}$$

$$BA = \begin{bmatrix} 3 & 2 \end{bmatrix}_{1 \times 2} \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{2 \times 1} = \begin{bmatrix} 3 \end{bmatrix}_{1 \times 1}$$

örnekler değişkenlik gösterebilir.